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COMMENT

On 'Exact solutions for the coagulation–fragmentation equation'

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Abstract. It is pointed out that the equilibrium solutions for pure coagulation and pure fragmentation which Dubovskii *et al* purport to obtain are in error, and that such solutions do not in fact exist.

In a recent paper Dubovskii *et al* (1992) discuss the existence of equilibrium and time-dependent solutions for the coagulation–fragmentation equation. The purpose of the present comment is to point out that the equilibrium solutions they purport to obtain for pure coagulation and pure fragmentation are in fact incorrect.

Let us consider first the case of pure coagulation, which is conventionally described by the standard equation

$$\frac{\partial c(m,t)}{\partial t}=\frac{1}{2}\int_0^m K(m-m_1,m_1)c(m-m_1,t)c(m_1,t)\,\mathrm{d}m_1 - \int_0^\infty K(m,m_1)c(m,t)c(m_1,t)\,\mathrm{d}m_1 \qquad (1)$$

in the notation of Dubovskii *et al.* Now, in each coagulation of a pair of clusters the total number of clusters decreases by unity, and thus it is impossible to find a solution for $c(m,t)$ which makes $\partial c(m,t)/\partial t=0$ for $m\geqslant 0$, $t\geqslant 0$, since such a solution would imply that the total number of clusters remains constant in time. In order to pinpoint the error in the work of Dubovskii *et al* let us consider a particular case of their conclusions corresponding to their equations (11) and (12) with $a=A=v(m)=1$. Their result is then that for $K(m,m_1)=(m+m_1)^{-3}$ there exists an equilibrium solution of equation (1) with $c(m,t)=1$. At first glance this would appear to be correct since each of the terms on the right-hand side of equation (1) then becomes equal to $1/(2m^2)$, and thus $\partial c(m,t)/\partial t=0$. The error here stems basically from the fact that equation (1) is strictly valid in the form stated only for $m>0$. For $m=0$ it takes the form

$$\frac{\partial c(0,t)}{\partial t}=-\int_0^\infty K(0,m_1)c(0,t)c(m_1,t)\,\mathrm{d}m_1 \qquad (2)$$

with the first term on the right-hand side omitted since no pair of smaller clusters can coagulate to produce a cluster of zero mass. The conventional form (1) for the equation is usually acceptable even for $m=0$ since the first term on the right-hand side is then automatically considered to be zero since both limits in the integral are the same (zero).

However in the work of Dubovskii *et al* leaving in this first term for $m=0$ gives rise to a contribution $\lim_{m\to 0}(\frac{1}{2}m^{-2})$ which is then considered to cancel out with minus this expression arising from the second term. In fact, only the latter expression should be present and since this is non-zero, then $\partial c(0,t)/\partial t \neq 0$ (in fact it is strictly negative). This in turn implies that the given form is *not* an equilibrium solution since this would require that $\partial c(m,t)/\partial t = 0$ for *all* $m \geqslant 0$, and as shown above this is not true for $m=0$.

Similar remarks to the above apply also the the pure fragmentation equation where the continual increase in cluster number again prevents the existence of an equilibrium solution. The detailed criticism of the mathematical treatment of Dubovskii *et al* follows similar lines to those of the previous paragraph.

Reference

Dubovskii P B, Galkin V A and Stewart I W 1992 *J. Phys. A: Math. Gen.* **25** 4737